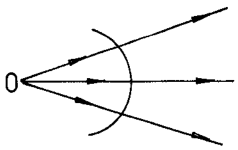

Fig 4. Concave spherical surface.

3 we can obtain the following relations:

$$\theta = \theta' + \theta_i - \theta_r$$

$$\psi = 90° + \theta - \theta_i$$

$$= \tan^{-1}\frac{dx}{dz}$$

$$r \sin\theta = r' \sin\theta'$$

$$\sin\theta_i = n \sin\theta_r \qquad \text{(Snell's law)}.$$

Thus,

$$\theta' = 90° + \psi + \sin^{-1}\left[\frac{1}{n}\cos(\theta - \psi)\right] \tag{5}$$

where n is the ratio of the refractive index of medium 2 to that of medium 1.

The radiation densities (power radiated per unit area) $S(\theta)$ and $S'(\theta')$ before and after refraction, respectively, are related to radiation intensities through

$$S(\theta) = r^2 P(\theta)$$

$$S'(\theta') = r'^2 P'(\theta').$$

Thus,

$$\frac{S'(\theta')}{S(\theta)} = \frac{r'^2}{r^2}\frac{\sin\theta}{\sin\theta'}\frac{d\theta}{d\theta'}.$$

The term $(DF)_2$ is the ratio of the electric field $U'(1)$ (just after refraction) to $U^i(1)$ (just before refraction). This ratio is given by

$$(DF)_2 = \frac{U'(1)}{U^i(1)} = \sqrt{\frac{S'(\theta')}{S(\theta)}}$$

or

$$(DF)_2 = \left(\frac{\sin\theta}{\sin\theta'}\right)^{3/2}\left(\frac{d\theta}{d\theta'}\right)^{1/2}. \tag{6}$$

The hyperboloidal surface of Fig. 2 is a special case where the angle θ' is equal to zero because of total collimation of the refracted rays and the term $(DF)_2$ can be shown to be

$$(DF)_2 \doteq \frac{(n\cos\theta - 1)^{5/2}}{F^2(n-1)^2(n-\cos\theta)^{1/2}}. \tag{7}$$

If the source is located at the center of a concave spherical surface as shown in Fig. 4, no refraction takes place because the rays are incident normally on the surface. Therefore the term $(DF)_2$ becomes unity. For surfaces other than spherical (Fig. 4) or hyperboloidal (Fig. 2) both $(DF)_1$ and $(DF)_2$ have values other than unity.

References

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Comments on "Improved Calibration and Measurement of the Scattering Parameters of Microwave Integrated Circuits"

ROGER MARKS

The above paper[1] proposes "generalized TRL" as an alternative to the TRL and LRL calibration methods. The contributions of the work, according to the authors, are "the reformulation in terms of S parameters and the removal of the requirement to specify a line length." In fact, it appears that *only* the formulation, not the method itself, is novel.

The original TRL method [1] utilizes a zero-length through connection. A more general calibration scheme, coined LRL [2], [3], replaces the through with a transmission line. The first stage of LRL is identical to TRL; the shorter line continues to be described *mathematically* as a zero-length through. As clearly pointed out by Hoer and Engen [2], [3], this results in calibration at a pair of "mating" references planes which coincide with the center of the short line. The second stage of LRL entails the movement of the reference planes back to the physical ports. It is *only* the movement of the reference planes that requires knowledge of the line lengths.

The current proposal is apparently just the first stage of LRL. As such, it avoids the need for line lengths solely by *leaving* the references planes in the center of the short line. This is demonstrated by the equivalence of the S parameters of the proposed method (equations (28)–(31) in the paper in question) with the analogous cascade coefficients of LRL (equations (1)–(6) of [2] or [3]). The proposed calibration scheme is not an "improved" or "generalized" form of TRL except to the extent that LRL is itself a generalization of TRL.

Furthermore, the authors' claim of a new method is unsupported by their experimental evidence, which offers only a comparison between their calibration and an *uncalibrated* test fixture.

Reply[2] by R. R. Pantoja, M. J. Howes, J. R. Richardson, and R. D. Pollard[3]

The comments raise four specific points which require some explanation in order to ensure proper understanding not only of what is described in our paper but also of the whole family of calibration procedures under the increasingly common TRL classification. First we must correct a misprint in our paper. In the first paragraph of Section III-A, the symbol Δl should be l_1, the length of the shorter line.

1) It must be emphasized that what is achieved in our paper is an S-parameter formulation of the TRL/LRL algorithm and a specific *application* to MIC characterization, neither of which has previously been presented in the literature.

2) In the context of the type of measurement discussed, the main issue is to locate suitably the calibration reference planes for measurement of a MIC structure while retaining the freedom of choice for lengths of both line standards and, consequently,

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[1]R. R. Pantoja, M. J Howes, J. R. Richardson, and R. D. Pollard, *IEEE Trans. Microwave Theory Tech.*, vol 37, pp. 1675–1680, Nov. 1989.
[2]Manuscript received December 8, 1989.
[3]The authors are with the Department of Electrical and Electronic Engineering, University of Leeds, Leeds, LS2 9JT, U.K.
IEEE Log Number 8934030.

for the location of the test port to which they will be connected during the calibration sequence. Contrary to the comment, TRL [1] should be viewed as a more general approach than LRL [2], [3] in that the latter is *restricted* to situations where both line lengths need to be *accurately* known. Still, the original TRL [1] appears to be somewhat restricted by the zero-length condition imposed on the shorter line. This is the reason to derive an *extension* of TRL, rather than LRL, so that no *accurate information* on the lines is needed. This is particularly desirable in an MIC environment, where it is difficult to determine accurate transmission line parameters and therefore the use of LRL as described in [2] and [3] is precluded.

3) Analysis of the original publication [2], [3] shows that the algorithm in our paper is *not* simply the first stage of LRL. The most important point to be noted is the fact that in [2] and [3] all three standards are connected to the analyzer *test ports* (see fig. 1 in [2] and [3]). Therefore, the estimate of the phase of the reflect is referenced to *these* ports. They are, by implication, the desired reference planes after calibration. Following [2] and [3], equations (1)–(6) express the error terms of an LRL calibration as a function of (i) the corresponding terms obtained by a TRL algorithm and (ii) the term $e^{\gamma l_1}$. What was not highlighted in [2] and [3] is the fact that A_{t_1} and A_{t_2} are known, at this stage, *except* for a sign ambiguity which is identified and accounted for in the paper in question because the reference planes are *wanted* at the center of the shorter line (see equations (19), (23–(27)). The same sign ambiguity in A_{t_1} and A_{t_2} does not affect the solution for the original LRL because only after the multiplication by $e^{\gamma l_1}$ (shift of reference planes towards the test port) is the reflect used to properly resolve it. This can be seen, accordingly, in equations (37)–(43) of our paper.

A word of caution is appropriate. If, as suggested in the comments, the shorter line is described as "mathematically zero-length," the estimate of the phase response of the reflect *ought* to account for this fact (by adding to it the phase through half of the shorter line l_1). In some situations, when γl_1 is less than 90° (l_1 sufficiently small), the resilience of the TRL algorithm will still provide the correct results. In conclusion, the error terms (equations (1)–(6) in [2] and [3]) will be equivalent to equations (28)–(31) in our paper, under all circumstances, *only* if the sign ambiguity in A_{t_1} and A_{t_2} is resolved. In the original LRL this is not necessary, but in applications where the reference planes are not at the test port this becomes imperative and only by detailed assessment of the technique can it be properly understood and implemented.

4) The experimental results in our paper show the effect of calibrations on in-fixture measurements. The movement of the reference planes from the 7 mm test port (trace *b* in the paper) to the MIC medium is accomplished by two different methods, *viz.* a "generalized TRL" calibration (trace *a*) and time-domain (gating) technique (trace *c*). Consequently, there are *no* uncalibrated test results—the data show the behavior from *different methods* of calibration.

Furthermore, the results demonstrate superior effective directivity and source/load match on microstrip than hitherto reported. The troughs in traces *a* and *c* are at the same frequencies, indicating that the reference impedance is very close to the Z_0 defined by the 7 mm calibration (50 Ω), corroborating the validity of the approach.

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